Hallgatói segédlet fizika BSc záróvizsga felkészüléshez

$$a = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos(\alpha_2 - \alpha_1)}, \quad \tan\alpha = \frac{a_1\sin\alpha_1 + a_2\sin\alpha_2}{a_1\cos\alpha_1 + a_2\cos\alpha_2} \tag{1}$$

$$A = \frac{a_0}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\beta^2\omega^2}}, \quad \tan\delta = \frac{2\beta\omega}{\omega_0^2 - \omega^2} \tag{2}$$

$$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{3}$$

$$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \tag{4}$$

$$T = 2\pi\sqrt{\frac{\Theta}{mgs}} = 2\pi\sqrt{\frac{l_r}{g}} \tag{5}$$

$$s = \frac{4}{E}\frac{l^3}{ab^3}F \tag{6}$$

$$\gamma = \frac{1}{G}\frac{F}{q} \tag{7}$$

$$\varphi = \frac{2}{\pi G}\frac{l}{R^4}M \tag{8}$$

$$\chi = 3\frac{1 - 2\mu}{E}, \quad G = \frac{E}{2(1 + \mu)} \tag{9}$$

$$p_g = \alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{10}$$

$$h = \frac{2\alpha\cos\vartheta}{\rho g r} \tag{11}$$

$$I = \frac{\pi}{8}\frac{1}{\eta}\frac{p_1 - p_2}{l}r^4 \tag{12}$$

$$R = \frac{\rho r v}{\eta} \tag{13}$$

$$\left(p + a\left(\frac{N}{V}\right)^2\right)(V - Nb) = NkT \tag{14}$$

$$R = c_p - c_v, \quad \kappa = \frac{c_p}{c_v} \tag{15}$$

$$c_V = \frac{1}{m}\left(\frac{dU}{dT}\right)_V \tag{16}$$

$$c_p = \frac{1}{m}\left(\frac{dH}{dT}\right)_p \tag{17}$$

$$\Delta T = \frac{\Delta p}{c_p}\left(\frac{2a'}{RT} - b'\right), \quad (a = a'N_A^2,\, b = b'N_A) \tag{18}$$

$$\eta = \frac{T_1 - T_2}{T_1} \tag{19}$$

$$\mathrm{d}S = \frac{\mathrm{d}Q_{rev}}{\mathrm{d}T} = \frac{\mathrm{d}U + p\mathrm{d}V}{T} \tag{20}$$

$$S = klnW \tag{21}$$

$$\frac{dT}{dp} = \frac{T(v_2 - v_1)}{L} \tag{22}$$

$$T_c = \frac{8a}{27bk}, \quad p_c = \frac{a}{27b^2}, \quad V_c = 3Nb \tag{23}$$

$$\overline{\xi^2} = \frac{kT}{3\pi\eta r}\tau \tag{24}$$

$$\sqrt{\overline{v^2}} = \sqrt{\frac{3kT}{\mu}} \tag{25}$$

$$\bar{l} = \frac{1}{4\sqrt{2}\pi r^2 N} \tag{26}$$

$$P(t) = U_{\text{eff}} I_{\text{eff}} (\cos\varphi - \cos(2\omega t - \varphi)) \tag{27}$$

$$U_{\text{ki,D}}(t) = U_0 \exp\left[-\frac{t}{\tau}\right] \tag{28}$$

$$U_{\text{ki,I}}(t) = U_0 \left(1 - \exp\left[-\frac{t}{\tau}\right]\right) \tag{29}$$

$$I_L = \frac{1}{L}\int U_0 \sin(\omega t) dt = I_0 \sin\left(\omega t - \frac{\pi}{2}\right) \tag{30}$$

$$I_C = \frac{dQ}{dt} = I_0 sin\left(\omega t + \frac{\pi}{2}\right) \tag{31}$$

$$IR + \frac{Q}{C} + L\frac{dI}{dt} = 0 \tag{32}$$

$$F = \frac{\mu_0}{4\pi}\frac{2I_1 I_2}{b} \tag{33}$$

$$B = \frac{\mu_0}{4\pi}\frac{q(\vec{v} \times \vec{r})}{r^3} \tag{34}$$

$$R = \frac{1}{q/m}\frac{v_\perp}{B} \tag{35}$$

$$m^* \frac{d\vec{v}_d}{dt} = -e\vec{E} - r\vec{v}_d \tag{36}$$

$$m = \frac{1}{F}\frac{M}{z}Q \tag{37}$$

$$\mathrm{d}q = \tau I \frac{\mathrm{d}T}{\mathrm{d}l} l \tag{38}$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t} \tag{39}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \tag{40}$$

$$\nabla \cdot \vec{D} = \rho \tag{41}$$

$$\nabla \cdot \vec{B} = 0 \tag{42}$$

$$\nabla\left(\frac{1}{\mu_0}\vec{E} \times \vec{B}\right) = -\frac{\partial}{\partial t}\left(\frac{1}{2}\frac{1}{\mu_0}B^2 + \frac{1}{2}\varepsilon_0 E^2\right) - \vec{E}\vec{J} \tag{43}$$

$$N_2(t) = N_1(0)\frac{\lambda_1}{\lambda_2 - \lambda_1}\left(e^{-\lambda_1 t} - e^{-\lambda_2 t}\right) \tag{44}$$

$$R = r_0 A^{1/3} \tag{45}$$

$$E(Z,A) = -e_b A + e_F A^{2/3} + e_C \frac{Z^2}{A^{1/3}} + e_A \frac{(A-2Z)^2}{A} + e_P \delta \frac{1}{A^{3/4}} \tag{46}$$

$$Z_{\min} = \frac{A}{2} \left(\frac{1}{1 + A^{2/3} \dfrac{e_C}{4e_A}} \right) \tag{47}$$

$$\frac{\partial^2 \Psi}{\partial t^2} = c^2 \left(\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} \right) \tag{48}$$

$$\Psi(x,t) = A \cdot \sin\left[\omega\left(t - \frac{x}{c}\right) + \alpha\right] \tag{49}$$

$$w = \frac{1}{2}\rho A^2 \omega^2 + \frac{1}{2}\rho A^2 \omega^2 \cos\left[2\omega\left(t - \frac{x}{c}\right) + 2\alpha\right] \tag{50}$$

$$I = c\overline{w} = \frac{1}{2}\rho c A^2 \omega^2 \tag{51}$$

$$\Delta E = \frac{E(Q)}{s} \cdot \sin\left[\omega\left(t - \frac{s}{c}\right) + \frac{\pi}{2}\right] \cdot K(\chi) \cdot \Delta S \tag{52}$$

$$\Psi(x,t) = 2A \cdot \cos\left(\frac{2\pi}{\lambda}x + \frac{\alpha_2 - \alpha_1}{2}\right) \cdot \sin\left(\frac{2\pi}{T}t + \frac{\alpha_2 + \alpha_1}{2}\right) \tag{53}$$

$$f' = \left(1 \pm \frac{v}{c}\right) f, \quad f' = \frac{f}{1 \mp \frac{v}{c}} \tag{54}$$

$$\sin\theta = \frac{v}{c} \tag{55}$$

$$J_\theta \approx \frac{\pi^2 (1 + \cos^2\theta)(n^2 - 1) V^2}{2 r^2 \lambda^4} J_0 \tag{56}$$

$$\Delta = \int\limits_{G_{AB}} n(\vec{r}) \mathrm{d}s \tag{57}$$

$$n = \frac{\sin\left(\dfrac{\delta_{\min} + \varphi}{2}\right)}{\sin\left(\dfrac{\varphi}{2}\right)} \tag{58}$$

$$N = s\left(\frac{1}{f} - \frac{1}{k}\right) \tag{59}$$

$$N = \frac{\Delta \cdot s}{f_1 \cdot f_2} \tag{60}$$

$$N = \frac{f_1}{f_2} \tag{61}$$

$$J = J_1 + J_2 + 2\sqrt{J_1 J_2} \cdot \cos\delta, \quad \delta = 2\pi \frac{n \cdot s_1 - n \cdot s_2}{\lambda} + \alpha_2 - \alpha_1 \tag{62}$$

$$l = c \cdot \Delta t \approx \frac{\lambda_0^2}{\Delta\lambda} \tag{63}$$

$$\Delta_{21} = 2dn\cos\beta = 2d\sqrt{n^2 - n_0^2 \sin^2\alpha} \tag{64}$$

$$I_t = \frac{I_0}{1 + F\sin^2\frac{\delta}{2}}, \quad F = \frac{4R}{(1-R)^2} \tag{65}$$

$$x_m^{(v)} = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n\theta}, \quad x_m^{(s)} = m\frac{\lambda}{2n\theta} \tag{66}$$

$$r_m \approx \sqrt{m\lambda\frac{ab}{a+b}}, \quad f_m = \pi\lambda\frac{ab}{a+b} \tag{67}$$

$$d\sin\alpha_1 = 1{,}22\lambda \tag{68}$$

$$r_\perp = \frac{\cos\alpha - n\cdot\cos\beta}{\cos\alpha + n\cdot\cos\beta} \tag{69}$$

$$t_\perp = \frac{2\cos\alpha}{\cos\alpha + n\cdot\cos\beta} \tag{70}$$

$$r_\parallel = \frac{n\cdot\cos\alpha - \cos\beta}{n\cdot\cos\alpha + \cos\beta} \tag{71}$$

$$t_\parallel = \frac{2\cos\alpha}{n\cdot\cos\alpha + \cos\beta} \tag{72}$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{2xy}{ab}\cos\delta = \sin^2\delta \tag{73}$$

$$u(\nu, T) = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{\frac{h\nu}{kT}} - 1} \tag{74}$$

$$\Delta\lambda = \lambda_C(1-\cos\vartheta), \quad \lambda_C = \frac{h}{mc} \tag{75}$$

$$E_n = -\frac{m_e e^4}{8h^2\varepsilon_0^2}\frac{1}{n^2} \tag{76}$$

$$a_0 = \frac{\hbar^2 4\pi\varepsilon_0}{e^2 m} \tag{77}$$

$$\alpha = \frac{e^2}{2\varepsilon_0 hc} \tag{78}$$

$$N_i^F = \frac{g_i}{Ae^{\frac{E_i}{kT}} + 1} \tag{79}$$

$$N_i^B = \frac{g_i}{Ae^{\frac{E_i}{kT}} - 1} \tag{80}$$

$$C_{e^-}(T) = \frac{\pi^2 N^{1/3}k^2 m}{(3\pi^2)^{2/3}\hbar^2}T = \frac{\pi^2 Nk^2}{2E_F(0)}T \tag{81}$$

$$\vec{b}_i = 2\pi\frac{\vec{a}_j\times\vec{a}_k}{\vec{a}_i\cdot(\vec{a}_j\times\vec{a}_k)} \tag{82}$$

$$V' = \frac{8\pi^3}{V} \tag{83}$$

$$\omega^2 = \beta\left(\frac{1}{m} + \frac{1}{M}\right) \pm \beta\sqrt{\left(\frac{1}{m} + \frac{1}{M}\right)^2 - \frac{4\sin^2 ka}{Mm}} \tag{84}$$

$$C_{\mathrm{Einstein}} = Nk\left(\frac{\Theta_E}{T}\right)^2 \frac{e^{\frac{\Theta_E}{T}}}{\left(e^{\frac{\Theta_E}{T}} - 1\right)^2}, \quad \Theta_E = \frac{\hbar\omega}{k} \tag{85}$$

$$C_{\mathrm{Debye}} = \frac{12\pi^4 Nk}{5}\left(\frac{T}{\Theta}\right)^3, \quad \Theta = \frac{\hbar\omega_{\mathrm{max}}}{kT} \tag{86}$$

$$m^* = \left(\frac{1}{\hbar^2}\frac{\mathrm{d}^2E(k)}{\mathrm{d}k^2}\right)^{-1} \tag{87}$$

$$E_{\mathrm{F}}(0) = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} \tag{88}$$

$$E_{\mathrm{F}}(T) = E_{\mathrm{F}}(0)\left[1 - \frac{\pi^2}{12}\left(\frac{kT}{E_{\mathrm{F}}(0)}\right)^2\right] \tag{89}$$

$$\chi_m = \frac{\mu_0 N \mu}{B} = -\frac{\mu_0 N Z e^2}{6m} < r^2 > \tag{90}$$

$$\chi_m = \frac{\mu_0 n \mu^2}{3kT} \tag{91}$$

$$\mu_B = \frac{e\hbar}{2m} \tag{92}$$

$$\vec{\mu} = -g\mu_B \vec{J} \tag{93}$$

$$H_i = -g\mu_B \vec{B}_0 \vec{S}_i - J \sum_{j'} \vec{S}_i \vec{S}_j = -g\mu_B \vec{S}_i \left(\vec{B}_0 + \frac{J}{g\mu_B} \sum_{j'} \vec{S}_j \right) \tag{94}$$

$$m_\infty = \frac{N}{V} \frac{g\mu_B}{2} \tag{95}$$

$$\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega) \tag{96}$$

$$\mu\varepsilon' = n^2 - k^2 \tag{97}$$

$$\mu\varepsilon'' = 2nk \tag{98}$$

$$\varepsilon'(\omega) = 1 + \frac{2}{\pi} P_f \int_0^\infty \frac{s\varepsilon''(s) ds}{s^2 - \omega^2} \tag{99}$$

$$\varepsilon''(\omega) = -\frac{2\omega}{\pi} P_f \int_0^\infty \frac{(\varepsilon'(s) - 1) ds}{s^2 - \omega^2} \tag{100}$$

$$A_{jk} = \frac{|p_{jk}^0|^2 \omega_{jk}^3}{3\pi\varepsilon_0 \hbar c^3} \tag{101}$$

$$B_{jk} = \frac{2\pi^2}{3\varepsilon_0 h^2} |p_{jk}^0|^2 \tag{102}$$

$$\mathrm{d}I = \frac{h\nu_{jk}}{c} (N_j B_{jk} - N_k B_{kj}) I \mathrm{d}x \tag{103}$$

$$z_R = \frac{\pi w_0^2}{\lambda}; \quad w(z) = w_0 \sqrt{1 + \left(\frac{z}{z_R} \right)^2}, \quad R(z) = z \left(1 + \left[\frac{z_R}{z} \right)^2 \right] \tag{104}$$

$$f(t) = a_0 + \sum_{k=1} (a_k \cos k\omega_0 t + b_k \sin k\omega_0 t) \tag{105}$$

$$a_0 = \frac{1}{T} \int_0^T f(t) dt \tag{106}$$

$$a_k = \frac{1}{T} \int_0^T f(t) \cos k\omega_0 t dt \tag{107}$$

$$b_k = \frac{1}{T} \int_0^T f(t) \sin k\omega_0 t dt \tag{108}$$

$$a_I(\omega) = \frac{1}{1 + jCR\omega} = \frac{1}{1 + j\frac{\omega}{\omega_p}} \tag{109}$$

$$a_D(\omega) = \frac{j\frac{\omega}{\omega_p}}{1 + j\frac{\omega}{\omega_p}} \tag{110}$$

$$\tan\varphi_I = -\frac{\omega}{\omega_p} \tag{111}$$

$$\varphi_D = \frac{\pi}{2} - \arctan\frac{\omega}{\omega_p} \tag{112}$$

$$\phi = \arctan\frac{\omega}{\omega_z} - \arctan\frac{\omega}{\omega_p} \tag{113}$$

$$I_D = I_S \left(e^{\frac{U}{U_T}} - 1 \right), \quad U_T = \frac{kT}{e} \tag{114}$$

$$\frac{1}{|\vec{r}-\vec{y}|} \approx \frac{1}{r} - \vec{y}\nabla\frac{1}{r} + \frac{1}{2}\sum_{i,j} y^i y^j \frac{\partial^2}{\partial x^i \partial x^j}\frac{1}{r}$$

$$\Phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} + \frac{\vec{p}\cdot\vec{r}}{r^3} + \frac{1}{6}\sum_{i,j} Q_{ij}\frac{\partial^2}{\partial x^i \partial x^j}\frac{1}{r}\right)$$

$$Q_{ij} = \int_V \rho(\vec{y})(3y^i y^j - y^2\delta_{ij})d^3\vec{y}$$

$$\mathrm{A}(\vec{r}) \approx \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

$$\vec{m} = \frac{1}{2}\int\limits_V \vec{y}\times\vec{J}\, d^3\vec{y}$$

$$\bar{B} = \frac{\mu_0}{4\pi}\frac{3(\bar{m}\bar{n})\bar{n} - \bar{m}}{r^3}$$

$$\bar{\nabla}\,\bar{A} = 0;\ \nabla\bar{A} + \frac{1}{c^2}\dot{\Phi} = 0$$

$$\Delta\psi(\bar{x},\bar{\omega}) + \mathrm{K}^2\Psi(\bar{x},\bar{\omega}) = -4\pi f(\bar{x},\bar{\omega})$$

$$\psi(\bar{x},\bar{\omega}) = \int_{\mathrm{V}} d^3x' f(x',\omega)G_k(\bar{x},\bar{x}')$$

$$G_K^{(\pm)}(R) = \tfrac{1}{R}e^{\pm i\mathrm{K}R}, \bar{R} = \bar{x} - \bar{x}'$$

$$\Delta\Psi(\bar{x},t) = \frac{1}{c^2}\ddot{\Psi}(\bar{x},t) = -4\pi f(\bar{x},t)$$

$$G_K^{(\pm)}(\bar{x},\overline{x'},t,t') = \frac{1}{2\pi R}2\pi\delta\left[t - \left(t \mp \frac{R}{c}\right)\right]$$

$$\Phi(\bar{x},t) = \frac{1}{4\pi\epsilon_0}\int d^3x'\frac{\rho(\bar{x}',t_r)}{R}$$

$$\bar{A}(\bar{x},t) = \frac{\mu_0}{4\pi}\int d^3x'\frac{\bar{J}(\bar{x}',t_r)}{R}$$

$$\frac{e^{i\kappa R}}{R} = \frac{e^{i\kappa r}}{r} - \frac{e^{i\kappa r}}{r}\left[i\kappa - \frac{1}{R}\right]\frac{\bar{x}-\bar{x}'}{r} + \sigma(|\overline{x'}|^2)$$

$$(\overline{A_1}(\bar{x},t) = -\frac{\mu_0}{4\pi}i\omega\bar{p}_1\frac{e^{i(\kappa r-\omega t)}}{r})$$

$$(\Phi(\bar{x},t) = (1-i\kappa r)\Phi_{dip}^{szt}(\bar{x})e^{i(\kappa r-\omega t)})$$

$$\bar{E}_1 = \left\{\frac{\kappa^2}{4\pi\epsilon_0}[(\bar{n}\times\bar{p}_1)\times\bar{n}]\frac{1}{r} + (1-i\kappa r)E_{dip}^{szp}\right\}e^{i(\kappa r-\omega t)}$$

$$\bar{B}_1 = \frac{\mu_0}{4\pi} i\omega(1 - i\kappa r)\frac{e^{i(\kappa r - \omega t)}}{r^2}\ \bar{n}\ \times \bar{p}_1$$

$$\bar{S} = \frac{\mu_0}{(4\pi)^2}\frac{\omega^4}{4}\frac{1}{r^2}\cos^2\left[\omega\left(t - \frac{r}{c}\right)\right] p_1^2 \sin^2\Theta\bar{n}$$

$$P = \frac{\mu_0}{12\pi}\frac{\omega^4}{c}p_1^2$$

$$\overline{R_V}(t') = R - \frac{\bar{R}\bar{V}}{c}$$

$$\overline{R_1} = \bar{R} - \frac{R\bar{V}}{c}$$

$$\Phi(\bar{x}, t) = \frac{q}{4\pi\epsilon_0}\frac{1}{R_v}$$

$$\overline{A_1}(\bar{x}, t) = \frac{q}{4\pi\epsilon_0}\frac{1}{c^2}\frac{v}{R_{v|t_{ret}}}$$

$$\bar{E} = \frac{q}{4\pi\epsilon_0}\left[\frac{R_1}{R_v^3}\left(1 - \frac{v^2}{c^2}\right) + \frac{1}{c^2}\frac{1}{R_v^3}\bar{R} \times (R_1 \times \bar{a})\right]$$

$$\bar{B} = \frac{1}{c}\frac{\bar{R} \times \bar{E}}{R}$$

$$\overline{S_{||}} = \frac{q^2}{16\pi^2\epsilon_0}\frac{a^2\sin^2\Theta}{c^3R^2}\bar{n}$$

$$P = \frac{q^2a^2}{6\pi\epsilon_0c^3}\ ; P = \frac{q^2}{6\pi\epsilon_0c^3m^2}\left[\left(\frac{d\bar{p}}{d\tau}\right)^2 - \frac{v^2}{c^2}\left(\frac{dp}{d\tau}\right)^2\right]$$

Vektoranalitikai képletek

$$\nabla \times \nabla f = \mathbf{0} \qquad \text{és} \qquad \nabla \cdot (\nabla \times \mathbf{A}) = 0$$

$$\begin{aligned}
\nabla(f\,g) &= (\nabla f)\,g + f\,(\nabla g) \\
\nabla \cdot (f\,\mathbf{A}) &= (\nabla f) \cdot \mathbf{A} + f\,\nabla \cdot \mathbf{A} \\
\nabla \times (f\,\mathbf{A}) &= (\nabla f) \times \mathbf{A} + f\,\nabla \times \mathbf{A}
\end{aligned}$$

$$\begin{aligned}
\nabla(\mathbf{A} \cdot \mathbf{B}) &= (\mathbf{A} \cdot \nabla)\,\mathbf{B} + (\mathbf{B} \cdot \nabla)\,\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A}) \\
\nabla \cdot (\mathbf{A} \times \mathbf{B}) &= \mathbf{B} \cdot (\nabla \times \mathbf{A}) - \mathbf{A} \cdot (\nabla \times \mathbf{B}) \\
\nabla \times (\mathbf{A} \times \mathbf{B}) &= \mathbf{A}\,(\nabla \cdot \mathbf{B}) - \mathbf{B}\,(\nabla \cdot \mathbf{A}) + (\mathbf{B} \cdot \nabla)\,\mathbf{A} - (\mathbf{A} \cdot \nabla)\,\mathbf{B}
\end{aligned}$$

$$\begin{aligned}
\nabla \times (\nabla \times \mathbf{A}) &= \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A} \\
\nabla^2(f\,g) &= f\,\nabla^2 g + 2\nabla f \cdot \nabla g + g\,\nabla^2 f \qquad [\Delta(f\,g) = f\,\Delta g + 2\nabla f \cdot \nabla g + g\,\Delta f]
\end{aligned}$$